Measures of Center and Variation Workshop Presentation Outline

Warm-up

1. What do you want your students to think about when they hear the term 'median'?
   Is this different from the actual meaning of the term?
2. What do you want your students to think about when they hear the term 'mean'?
   Is this different from the actual meaning of the term?
3. How do we represent the 'spread' of a data set?

"average" versus "mean"

"Developing a Meaningful Understanding of the Mean"

When introduced, mean is often presented as an application of division rather than a statistical concept

* Possibility exists for students to <u>compute the mean without developing an understanding</u> of what the mean represents, how it is related to the numbers in the data, or how it is related to other measures of the center of spread of a data set – as a procedure
* Seven properties of the mean
  1. The mean is located between extreme values
  2. The sum of the deviations from the mean is zero
  3. The mean is influenced by values other than the mean
  4. The mean does not necessarily equal one of the values that was summed
  5. The mean can be a fraction that has no counterpart in physical reality
  6. When one calculates the mean, a value of zero, it if appears, must be taken into account
  7. The mean value is representative of the values that were averaged
* 'fund-raising contest' problem
  Best performance amongst groups of different size
  Discussion of mean and range
  Sum of deviations from mean equal 0

'what happens if . . .?' problem

Problem without context, asks students to develop
Mean value can have no meaning in context
How adding values to a data set affects the mean

Measures of Center and Variation Workshop Presentation Outline

NCTM Professional Development experience from the web
Active research

Follow-up

How is this represented in grades 6-12?

"Highlights of *PSSM* Data Analysis and Probability Standard"
Purpose and meaning of using mean and median: Can it be misleading?
What characteristics of value changes affect mean & median?
Exploration of categorical data

Women's Soccer Results: Exploring the Mean
Identify three types of average: mean, median, mode
Mean as a balance point of a distribution
Create data sets for a given mean
Estimate mean of small sets of data
Cubes to represent data values on line plot, shift around
Post-It Notes along a scale line

Fathom "Meaning of Mean"

"Children's Concepts of Average and Representativeness"
* Educators assume average is simply another application of division and if children understand fair or equal share, they will understand the notion of average
Examine how child describes and constructs sets of data while examining how they understand average
Students develop their own impression of 'average' & typical
Two research questions (n = 21)
When asked to describe a real data set, how do children construct and interpret representativeness?
How do children understand this mathematical objet and how do they connect it with their informal mathematical understanding?
Student interviews and think aloud to explain answers

"CHILDREN'S CONCEPTS" CONTINUED

* IDENTIFY FIVE PREDOMINANT APPLICATIONS TO SOLVE PROBLEMS
  1. AVERAGE AS MODE
  2. AVERAGE AS ALGORITHM
  3. AVERAGE AS REASONABLE
  4. AVERAGE AS MIDPOINT
  5. AVERAGE AS MATHEMATICAL POINT OF BALANCE
* LAST THREE APPLICATIONS INDICATE STUDENTS WORKING TOWARD A DEFINITION OF AVERAGE

  PREMATURE LEARNING OF THE ALGORITHM HURTS CONCEPTUAL UNDERSTANDING OF AVERAGE

UNDERSTANDING THE MEAN AND THE MEDIAN THROUGH AN INTERACTIVE ACTIVITY

HOW CHANGE(S) IN DATA VALUE(S) AFFECT MEAN/MEDIAN

WHY?

APPLET (STANDARDS CD – SECTION 6.6)

WHAT SORT OF DATA CHANGES AFFECT THE MEAN? THE MEDIAN?

DESIGN DATA SETS TO GIVEN MEAN AND MEDIAN

HOW MANY POSSIBLE?

CAN YOU FIND WAYS TO MOVE DATA POINTS TO KEEP THE MEAN/MEDIAN THE SAME BUT CHANGE THE MEDIAN/MEAN?

"MEAN AND MEDIAN: ARE THEY REALLY SO EASY?"

* PROCEDURAL VS CONCEPTUAL UNDERSTANDING

  STUDENTS TEND TO SELECT MEAN OVER MEDIAN TO REPRESENT A DATA SET REGARDLESS OF DISTRIBUTION

  NAEP ITEMS: MEAN/MEDIAN/MODE, MEDIAN FROM SCATTERPLOT, BEST MEASURE
* IS AVERAGE BETTER UNDERSTOOD THAN MEAN?

  IDEA THAT MEAN IS MORE PRECISE/ACCURATE BECAUSE IT INCLUDES ALL OF THE NUMBERS AND MEDIAN IS JUST ONE NUMBER

FATHOM "TRANSFORM MEAN AND STANDARD DEVIATION"

"NORMAL DATA"

STANDARD DEVIATION IS $\sqrt{\sum(\text{distance from mean})^2}$

Measures of Center and Variation Workshop Presentation Outline

A Balancing Act *Student Math Notes* activity

- Examining how the mean of a data set is the same as the balancing point along a scale
- Expands to examine the concurrence point of the medians of a triangle is also the balancing point
- Includes the weighting of points

"What do children Understand about Average?"

* Understand procedure but not concept
* Creating data set with given average
  - Students tend to select a single value (mode view)
  - Some used average as middle and then 'balanced' symmetrically (median view)
  * Frustrated if tried to use algorithm for mean and work backwards
  - Students need to understand median before mean
* "Fair Share" model had problem of losing the original elements of data
  - Connect to Soccer Goals activity

"What do children Understand about Average?" *continued*

- "Balance" model looks at differences from mean, but student must understand the concept of balance
- "Unpacking" model uses line plots and post-it notes, incorporates idea of balance
- Importance of context and multiple examples
- Action research ideas offered

A Look at the Average Wage

- Mean, median, and mode from frequency table
- Changing data values and reflected change in measures of central tendency
- Spreadsheet extension
- Using measures of central tendency for arguments

TI-84 to find mean and median from frequency table

- Data in L1, Frequency in L2, then 1-Var Stats L1, L2

Measures of Center and Variation Workshop Presentation Outline

"Teaching Statistics – What's Average?"

* Mode – some students confuse 'most frequent' with 'most have'
  - Only measure of center than can be categorical
  - Not particularly useful with quantitative data
* Median – placing data elements on grid paper and folding in half to represent as the middle of an organized data set
  - Moving to abstract thinking process
  - Stability of the statistic
* Mean – Fair share/evening out strategy or using the balance model
* Best use activity – when and why (contextual)

TV Watching

Do middle school students watch too much TV?

Mean/median/mode, perspective (student and parent), subjective Justification

Drop Off

Interpret histogram, identify measure of central tendency, make inferences

Sampling Rectangles

Area of 'typical' rectangle vs 'random' rectangle

Sampling distributions, simple random sample, variability, bias

Measures of center and spread

Types of samples and responses Sample vs population

Extension: **Math Phobia Media Clips**

Total time of train travel with a 10% reduction in speed after every stop

Create data sets with mean = median, mean > median, and mean < median

Mean and median incomes with groups of people involving Bill Gates

Median as 'midpoint'

Extension: **Position of the Geographic Center** of Area, Mean, and Median Centers of Population

Extension: **Calendar Square Mean**